Seakeeping Dynamics of a Single Cushion, Peripheral Cell-Stabilized Air Cushion Vehicle

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A study of air cushion vehicle (ACV) motion in waves is presented for a single cushion ACV having a cellular, peripheral cell-type skirt system. The craft is considered to be traveling at constant speed while encountering regular waves of arbitrary heading. The dynamic equations for pitch, heave, and roll motions are derived using the cushion and cell air flow equations. These equations are solved numerically using a digital computer. The results are shown as frequency response curves giving steady-state motion response amplitudes as a function of encounter frequency or wavelength for fixed craft speed and wave steepness. The theoretical predictions are then compared with experimental data taken from scale model, towing tank tests in head seas. The comparison shows good agreement for pitch motion, while heave motion damping is overpredicted.

Nomenclature

a	= wave amplitude
A_c	= cushion planform area
$(A_j)_i$	= area of jupe (cell) bottom opening
A_L	= leakage area
$(A_L)_i$	= area of orifice between loop and vertical cell
A_0	= equilibrium leakage area
b	= beam
C_n	= orifice coefficient
f	= cushion leakage fraction of gap height
h_i	= depth below origin of fully extended skirt at ith jupe
H	= unit step function
I_x	= craft moment of inertia about the x axis
I_y	= craft moment of inertia about the y axis
K	= roll moment
k	= wavenumber
k_1,k_2	= increase in cell width on inside, outside per unit change in height
l	= cushion length
M	= pitch moment
m	= mass of craft
m_c	= mass of air in cushion
P	= cushion gage pressure
P_L	= loop pressure
Q	= volume rate of flow
$Q_{0,1,2}$	= fan performance parameters
ΔS_i	= length of skirt seal around cushion periphery
T	= effective width of skirt jupes
U_0	= craft speed
V_c	= cushion volume
W	= craft weight
x_c	= x coordinate of cushion centroid
z_c	= depth of top of cushion from origin
z_0	= heave coordinate
Z	= force on craft in the z direction
β_i	= angle of outward normal at the ith cell with respect to the x axis
γ	= angle of wave propagation direction with respect to the x axis
γ_c	= ratio of specific heats ($=c_p/c_v$)
η	= wave height
θ	= pitch angle
λ	= wavelength
ρ	= density
ϕ	= roll angle
ω_e	= encounter frequency
ω	= wave frequency

Subscripts

a	= refers to atmosphere
i,j	= refers to ith, jth seal (jupe)
L	= refers to loop
0	= refers to equilibrium conditions

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I. Introduction

THE problem considered in this paper is the seakeeping dynamics of an air cushion vehicle (ACV) in regular waves of arbitrary heading. More specifically, the pitch, heave, and roll motions are predicted theoretically for a single cushion peripheral cell-stabilized craft as it travels at constant speed and course through sinusoidal, noncompliant waves.

The air cushion vehicle to be discussed has a skirt configuration which consists of a single main cushion surrounded at the craft periphery by a flexible skirt made up of a series of vertical cells (see Fig. 1). Air is fed through orifices to the cells or jupes from a loop plenum located above the jupes and extending around the perimeter of the vehicle. The air supply inflates each cell and then escapes through the open bottom forming a vertical jet. The loop plenum and the main cushion are each supplied with air from separate fan systems. The cellular skirt arrangement provides pitch and roll stability. Also, the downward jet flow beneath the skirt around the edge of the cushion forms a "curtain" effect minimizing leakage from the main cushion.

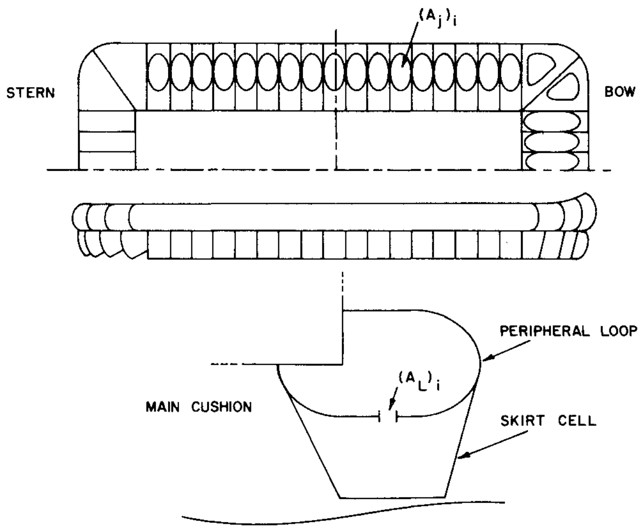

Fig. 1 Skirt configuration.

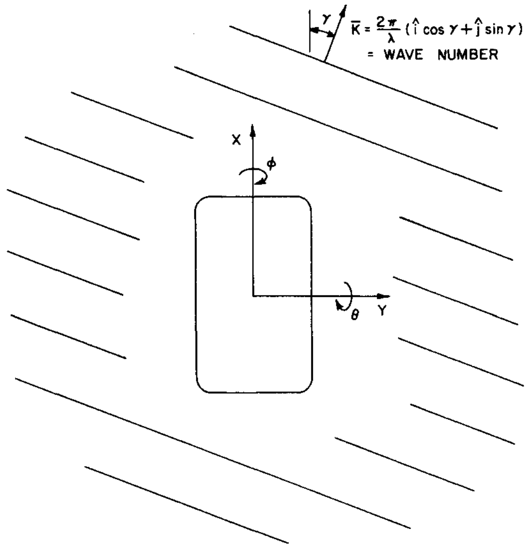

Fig. 2 Craft coordinate system.

A mathematical model is developed for the dynamics of this craft in waves by considering the vehicle rigid body equations of motion, the conservation of air mass and flow relations, and the adiabatic gas law. These governing nonlinear dynamic equations are solved numerically using a digital computer. The analysis is then applied to a particular craft using both full-scale design particulars and those for a 7/100 scale model. The predictions for the scale model motion as the craft encounters head seas are compared with experimental data from seakeeping tests made in the towing tank. Thus, a check is provided on the validity of the theoretical model.

Previous work on the theoretical prediction of the seakeeping of air cushion vehicles which has been reported in the literature includes an analysis of the vertical plane dynamics of SES craft by Kaplan and Davis.[1] They presented a simplified approach to the problem enabling some discussion of natural frequencies and damping ratios to be made before resorting to computer simulation. Lavis et al.[2] considered the response of a peripheral cell air cushion vehicle in random seas using a model which used empirical data for the roll and pitch stiffness and damping. In addition, they discussed atmospheric scaling and the consequent problem of lift system parameter distortion in scale models by using an analysis of the flow equations associated with heave motion. Linear models for hovercraft motion in regular waves were also developed by Reynolds[3] and Reynolds et al.[4] A nonlinear model for the coupled pitch and heave motion of an air cushion vehicle in regular waves was developed by Doctors.[5] The craft he considered was of the divided cushion type in which the main cushion is divided into subcushions by flexible longitudinal and transverse stability keels. More recently, Doctors has extended his analysis of divided cushion craft to include the effects of hydrodynamic influence and compressibility.[6,7] Carrier et al.,[8] on the other hand, have presented the results of a nonlinear analysis for the coupled pitch and heave motion for both a divided cushion craft and a vehicle of the single cushion, peripheral cell type. All of the aforementioned nonlinear models required a computer-evaluated numerical solution of the dynamic equations.

In related experimental investigations, Magnuson[9] and Fein et al.[10] reported full-scale seakeeping trial data for the performance of a 50-ton British Hovercraft. Magnuson and Mesalle[11] have also used the results of model experiments to develop linear equations of motion for the JEFF (B) in head waves.

In this paper a nonlinear model for pitch, heave, and roll motions in waves is developed from first principles for a peripheral cell ACV. The mathematical model of the equations of motion is then solved on a digital computer using numerical techniques. This is the first simulation of a peripheral cell ACV that takes into account the stabilizing effect of the peripheral cells from first principles. (The analysis in Ref. 2 used a linear model for pitch and roll where the natural frequencies and damping ratios were obtained empirically.) That is, the peripheral cells are modeled by solving the flow equations for each cell, computing the resulting cell pressures, and using these in the equations of motion after computing the pressure forces and moments. Another unique feature of this simulation is the way the leakage area for the main cushion is computed. Since leakage from the loop-skirt system as well as the main cushion must exit through the geometrical gap between the outer skirt and the water surface, a reduced gap height is used to calculate the main cushion leakage area. The exit area is, in effect, partitioned between the area available for main cushion escape flow and that for the seal system. The reduction in gap height is related to cell and cushion flow rates at equilibrium.

A simulation developed from first principles is most valuable for the vehicle designer, as extensive parametric design studies can be conducted that would not be economically feasible using model experimental techniques. The simulation must, of course, be compared to model data and/or trial data to ascertain its accuracy, and to provide guidance for future efforts in improvement of the simulation. A comparison is therefore made between the simulation predictions and model motion data taken in regular head seas.

II. Theoretical Model

The position and orientation of the ACV is given by the heave coordinate z_0 (positive downward), pitch angle θ, and roll angle ϕ. An x, y, z coordinate system, fixed with respect to the craft is located so that its origin is at the center of mass (see Fig. 2). The craft, traveling at constant speed U_0, encounters regular waves propagating at an angle γ to the vessel's course. Viewed from the craft, then, the surface wave field is given by

$$\eta = a\sin\{2\pi/\lambda[(\cos\gamma)x + (\sin\gamma)y] - \omega_e t\}$$

where

$$\omega_e = \text{encounter frequency} = \omega - (\omega^2 U_0/g)\cos\gamma \tag{1}$$

The equations of motion governing the craft response are

$$m\ddot{z}_0 = (Z)_{\text{cushion}} + (Z)_{\text{seals}} + W \tag{2a}$$

$$I_y \ddot{\theta} = (M)_{\text{cushion}} + (M)_{\text{seals}} \quad (2b)$$

$$I_x \ddot{\phi} = (K)_{\text{cushion}} + (K)_{\text{seals}} \quad (2c)$$

The cushion and seal forces and moments will be specified in terms of craft position and cushion and cell pressures.

The main cushion pressure must be such that the rate of change of the cushion mass equals the net rate of mass flow into the cushion. This is expressed by

$$\frac{dm_c}{dt} = \rho_{\text{in}} Q_{\text{in}} - \rho_{\text{out}} Q_{\text{out}} \quad (3)$$

It will be assumed that the air density while entering and leaving the cushion is approximately the atmospheric air density.

$$\rho_{\text{in}} \simeq \rho_{\text{out}} \simeq \rho_a \quad (4)$$

The flow into the cushion is due to the fan air supply and is expressed in terms of the cushion pressure

$$Q_{\text{in}} = Q_0 + Q_1 P + Q_2 P^2 \quad (5)$$

where the coefficients are determined empirically, Equation (5) approximates the "fan curve" for the supply fans.

The flow out of the main cushion is due to leakage between the skirt and the water surface and is specified using an orifice coefficient modification to the Bernoulli equation:

$$Q_{\text{out}} = C_n A_L \sqrt{2P/\rho_a} \quad (6)$$

The total leakage area A_L consists of an equilibrium leakage area A_0 and leakage area due to the physical gap between each element of the skirt and the surface,

$$A_L = A_0 + f \sum_{\substack{\text{craft}\\ \text{periphery}}} G_i \Delta S_i, \qquad A_0 = \frac{(Q_{\text{in}})_0}{C_n \sqrt{2(P)_0/\rho_a}} \quad (7a)$$

$$G_i = (-z_0 - h_i + x_i\theta - y_i\phi - \eta_i) H(-z_0 - h_i + x_i\theta - y_i\phi - \eta_i) \quad (7b)$$

The gap height at the ith cell, G_i is, however, not all available for main cushion leakage. Air from the underfed peripheral jets directed downward beneath the cells must also escape under the outer hem of the skirt. That is, the actual flow through the physical gap between the outer skirt and the water surface consists of flow from the main cushion and deflected jet flow from the cellular skirt. The effective gap height for the main cushion is then a fraction f of the physical gap height. Due to the relatively large openings at the base of the open cells, the cells do not function as effective nozzles, and the downward flowing air is immediately diverted horizontally outward. This allows the apportionment of escape areas at equilibrium to be determined by conservation of mass considerations which imply that the fraction f is equal to the ratio of equilibrium cushion supply to total fan feed. Since the local behavior of escape flows is similar when the craft is in a dynamic situation, the fraction f is approximated by the value known to exist at equilibrium.

The rate of change of the mass of air in the main cushion is expanded by writing

$$\frac{dm_c}{dt} = \rho \frac{dV_c}{dt} + V_c \frac{d\rho}{dt} \quad (8)$$

The cushion volume depends on the craft position relative to the waves:

$$V_c = A_c(-z_0 + x_c\theta - z_c) + \frac{\lambda^2 a}{\pi^2 \cos\gamma \sin\gamma} \times \sin\frac{\pi l \cos\gamma}{\lambda} \sin\frac{\pi b \sin\gamma}{\lambda} \sin\omega_e t \quad (9)$$

The first term in the preceding expression is due to changes in average height of the cushion, while the second term represents the wave "pumping" effect.

Compressibility effects of the cushion air mass are taken into account by the last term in Eq. (8). The cushion density is related to cushion pressure by assuming that the thermodynamic processes are reversible and adiabatic. The rate of density change may then be expressed by

$$\frac{d\rho}{dt} = \frac{1}{\gamma_c} \rho^{1-\gamma_c} \left(\frac{\rho_a^{\gamma_c}}{P_a} \right) \frac{dP}{dt} \quad (10)$$

Thus, Eqs. (3-10), which represent conservation of mass for the main cushion air flow, relate the cushion pressure to craft position, surface wave motion, and the rates of motion of the vehicle.

The cushion forces on the craft are the resultants of the cushion air pressure acting over the area of the cushion supported externally. The z-direction force is

$$(Z)_{\text{cushion}} = -PA_C \quad (11)$$

while the pitch and roll moments are

$$(M)_{\text{cushion}} = \sum_i (-\cos\beta_i) P[\eta_i - x_i\theta + y_i\phi] \Delta S_i (-z_0 - \eta_i) - x_c (Z)_{\text{cushion}} \quad (12)$$

and

$$(K)_{\text{cushion}} = \sum_i (\sin\beta_i) P[\eta_i - x_i\theta + y_i\phi] \Delta S_i (-z_0 - \eta_i) \quad (13)$$

respectively. The moment expressions include contributions due to components of the resultant, externally applied cushion force lying parallel to the plane of the craft as well as the z-direction component.

The seal forces will be specified by considering each jupe or cell in a manner roughly analogous to the treatment of the main cushion. The open cells themselves may be thought of as a series of minicushions, contained by completely flexible seals, positioned around the bow and sides of the craft. For the ith cushion, then, conservation of mass is required for the air flow through the jupe. This would be a statement that the volume rate of flow into the cell from the loop plenum must equal the flow out the cell bottom opening plus the rate of cell volume increase. In equation form, this becomes

$$C_n (A_L)_i \sqrt{\frac{2|P_L - (P_j)_i|}{\rho_a}} \operatorname{sgn}[P_L - (P_j)_i] = C_n [(A_{0_i}) + (1-f)\Delta S_i(-z_0 + x_i\theta - y_i\phi - h_i - \eta_i)] H[(A_0)_i + (1-f)\Delta S_i(-z_0 + x_i\theta - y_i\phi - h_i - \eta_i)] \sqrt{\frac{2(P_j)_i}{\rho_a}} + T\Delta S_i(-\dot{z}_0 + x_i\dot{\theta} - y_i\dot{\phi} - \dot{\eta}_i) H(z_0 - x_i\theta + y_i\phi + h_i + \eta_i) \quad (14)$$

where the term on the left-hand side represents the flow into the cell through the orifice connecting the loop to the cell. The first and second terms on the right-hand side account for escape flow under the outer seal and the cell volume rate of change, respectively. The escape area for the leakage flow is made up of a portion of the geometrical gap and leakage area present at equilibrium, $(A_0)_i$. Since the equilibrium jupe pressure is the same as for the main cushion, $(A_0)_i$ can be expressed in terms of the cell's equilibrium flow rate $(Q_i)_0$ by

$$(A_0)_i = \frac{(Q_i)_0}{C_n\sqrt{2P_0/\rho_a}} \tag{15}$$

For the closed jupes across the stern, the second term in Eq. (14) is omitted since there is no escape flow out the bottom.

Though the loop plenum is fed separately from the main cushion, the gap exit flow areas for the cushion and the loop-skirt system are proportional. The flow out from the two systems is also nearly proportional, since the pressure of each cell is usually very close to the cushion pressure. Because the total flow through the loop-skirt system is nearly proportional to flow through the cushion, the loop pressure will be assumed proportional to the cushion pressure. The constant of proportionality will be taken as the ratio of the respective pressures at equilibrium. With this approximation and conservation of mass, the pressure within each cell may be related to cushion pressure, craft motion, and wave motion.

The additional vertical force (above that already included in the main cushion forcing term) on the ith seal Z_i is obtained using the assumption that the seal material itself is completely flexible and without inertia. The force Z_i is then equal to the difference in pressures (between the cell and cushion) acting over the externally supported, cell base area within the cushion planform plus the cell pressure acting over an area increase outside the equilibrium planform:

$$Z_i = -[(P_j)_i - P](A_j)_i + \{-[(P_j)_i - P](k_1)_i[z_0 - x_i\theta + y_i\phi + h_i + \eta_i] - (P_j)_i(k_2)_i[z_0 - x_i\theta + y_i\phi + h_i + \eta_i]\}\Delta S_i H(z_0 - x_i\theta + y_i\phi + h_i + \eta_i) \tag{16}$$

The resultant seal forces and moments are then

$$(Z)_{\text{seal}} = \sum_i Z_i,\quad (M)_{\text{seal}} = \sum_i (-x_i)Z_i,\quad (K)_{\text{seal}} = \sum_i (y_i)Z_i \tag{17}$$

III. Solution of Equations of Motion

The dynamic equations obtained from the theoretical model are solved numerically using a digital computer. After experimenting with several algorithms for solving differential equations, it was found that a polynomial extrapolation method (taken from Gear[12]) was the best means for extrapolating time-dependent variables from one time step to the next. In the simulations, the craft trajectories were calculated as the vehicle ran at constant speed through regular waves. For different runs, various wavelengths, speeds, waveheights, and wave propagation directions were specified. As the craft approached steady-state motion, the time response was Fourier analyzed to determine its harmonic constituents. In particular, the magnitude and phase of the first harmonic response of the time-dependant variables was determined for a range of encounter frequencies.

This procedure was applied to a specific craft having a skirt configuration of the single cushion, trunked peripheral jet type. The planform of the example considered is taken to be rectangular. Craft particulars are listed in Table 1.

The numerical simulation was also applied to a 7/100 scale model of the vehicle. The parameters for this case were Froude scaled from the full-sized craft. The scale model analysis was done for the purpose of comparing, at the model scale, the theoretical analysis with towing tank experiments for a similar model. The empirical data were obtained from a series of tests, conducted at the David W. Taylor Naval Ship Research and Development Center, on a physical scale model of the AALC JEFF(A). This model is similar to the example chosen for numerical simulation except that the physical model has a rounded bow. In addition, the fan curve used in the simulation differed from the model. (Because of time limitations, it was not possible to implement the more realistic cushion planform shape and fan curve.)

Table 1 Craft physical parameters

W	$= 320{,}000$ lb
I_y	$= 4.75 \times 10^6$ ft-lb-s^2
I_x	$= 1.21 \times 10^6$ ft-lb-s^2
l	$= 82.2$ ft
b	$= 42$ ft
h_i	$= 10$ ft
x_c	$= 0$ ft
z_c	$= 5$ ft
β_i	$= +90$ deg, -90 deg along starboard, port side of craft; 0 at bow; 180 deg at stern
k_1	$= 1$ along bow and sides; 4 at stern
k_2	$= 0.6$ along bow and sides; 4 at stern
ΔS_i	$= 4.33$ ft along sides (19 cells on each side) 5.25 ft across bow and stern (8 cells on bow, stern)
T	$= 8.0$ ft
Q_0	$= 4200$ ft^3/s
Q_1	$= -11$ ft^5/lb-s
Q_2	$= 0$
P_a	$= 2120$ lb/ft^2
γ_c	$= 1.4$
ρ_a	$= 2.2 \times 10^{-3}$ slug/ft^3
$(P_L)_0/(P)_0$	$= 1.3$
C_n	$= 0.7$
f	$= 1/4$
$(A_L)_i$	$= 2.5$ ft^2
$(A_j)_i$	$= 20$ ft^2 for bow and sides; 0 at stern

IV. Results

The results are shown as frequency response plots in Figs. 3-9. These plots show the nondimensional steady-state amplitude and phase of the motion response for the craft running at a constant speed and heading in regular sinusoidal waves. The frequency response plots were obtained by making successive runs on the simulation for a fixed speed and heading and analyzing the response digitally to obtain the first harmonic or fundamental component (both amplitude and phase) from the response after the initial transients had decayed out. The results are plotted as a function of the encounter frequency, which varies with the wavelength for a fixed heading and speed. All of the results shown are for a wave with a constant steepness ($2a/\lambda = 1/80$). This was the nominal wave steepness used in the model experimental motions program. The results are presented in nondimensional form in a manner consistent with Froude scaling.

Comparing the predicted model first harmonic curve with the experimental curve in Fig. 3, it is seen that the model data indicate a slightly underdamped response, while the predicted heave response is overdamped. The break frequency or cutoff frequency, however, corresponds well. A somewhat similar situation is found at the higher speed of 50 knots shown in Fig. 4: slightly underdamped model response and overdamped predicted response.

For the pitch motion, on the other hand, both the predicted pitch motion and the experimentally observed pitch are definitely underdamped. At 35 knots (Fig. 5) the damping is very close, but the natural frequency is overpredicted by about 20%. At 50 knots, however, Fig. 6 shows very good agreement for both the pitch damping and the natural frequency.

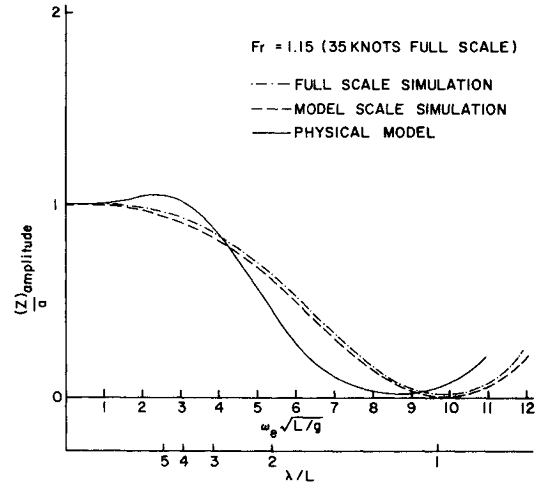

Fig. 3 First harmonic response for heave in head seas ($F_r = 1.15$).

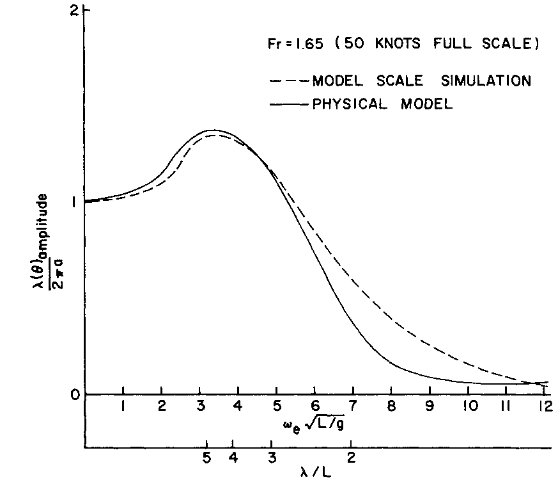

Fig. 6 First harmonic response for pitch in head seas ($F_r = 1.65$).

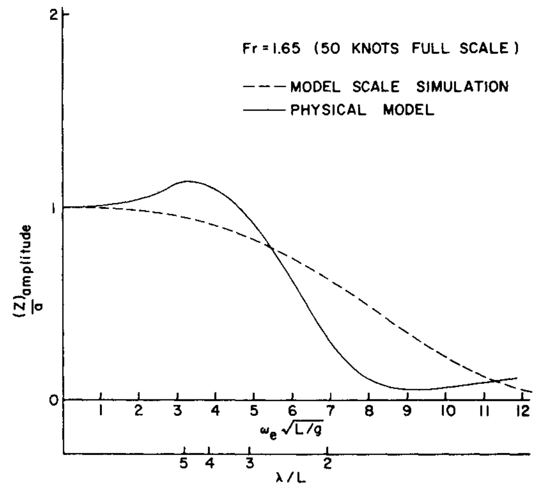

Fig. 4 First harmonic response for heave in head seas ($F_r = 1.65$).

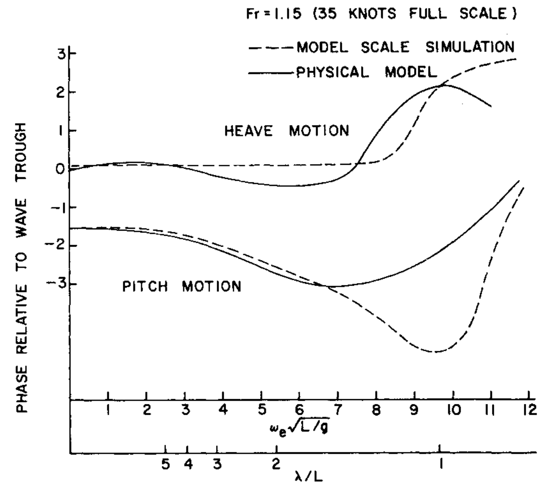

Fig. 7 Phase angles of craft response ($F_r = 1.15$).

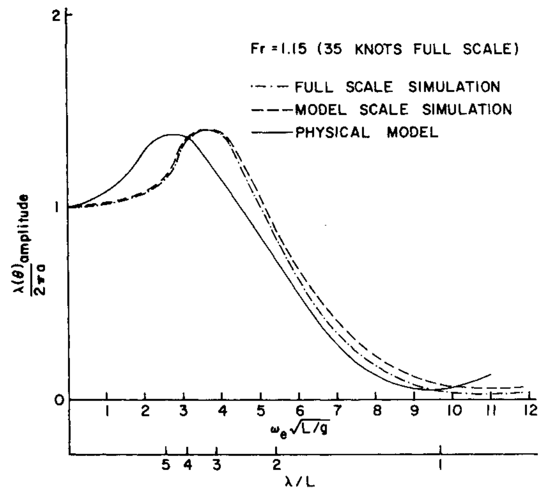

Fig. 5 First harmonic response for pitch in head seas ($F_r = 1.15$).

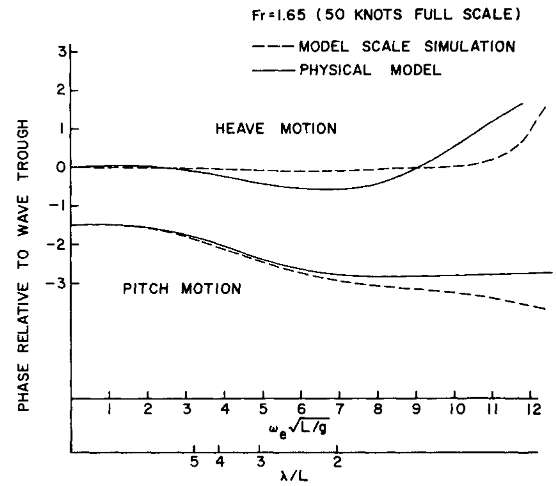

Fig. 8 Phase angles of craft response ($F_r = 1.65$).

Phase angle comparisons are shown in Figs. 7 and 8 for $F_r = 1.15$ and $F_r = 1.65$, respectively. The phase angles are taken relative to the wave profile at the longitudinal center of gravity. One sees good agreement with experiment for both speeds, except at the higher frequencies where the motions are attenuated. The low-frequency phase angle in heave approaches 0 and the pitch phase approaches $-\pi/2$ rad, indicating that the craft tends to contour the longer waves in both pitch and heave. (The wave slope lags the wave height by $-\pi/2$ rad because of the positive downward sign convention on the wave profile.)

In connection with these comparisons, it should be mentioned that the experiments have suggested that the heave resonances were due to a coupling between heave and pitch motion as a result of fore and aft asymmetry. This could explain the lack of complete agreement in heave, since sources

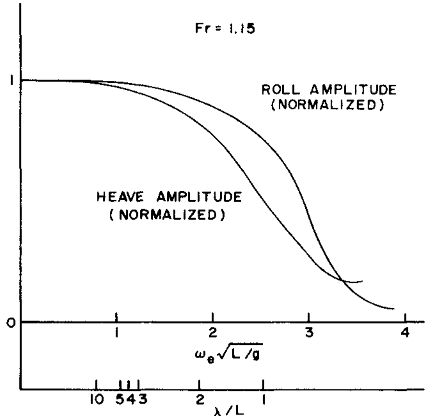

Fig. 9 Response to beam seas.

of asymmetry not included in the numerical simulation were present in the physical model. For example, geometrical asymmetry due to the rounded bow and asymmetry of the wave field caused by the vehicle's own wave patterns were not incorporated in the mathematical model. In addition, the inaccurate fan curve may have accounted for some of the discrepancy in heave.

A comparison of the curves in Figs. 3 and 5 for the full-scale craft with those predicted for the model scale shows that, according to the simulation, scale effects resulting from ignoring pressure scaling are relatively unimportant. This implies that the importance of atmospheric scaling is much less than has been previously thought and that direct extrapolations of model data to full scale using Froude scaling should be permissible within the accuracy indicated by the results. It should be noted that the fan curve for the full-scale simulation was obtained by a direct Froude scaling of the model fan curve. That is, no adjustment was made between the model and full-scale fan slope to compensate for differences in air cushion stiffnesses.

In conclusion, the nonlinear simulation model represented here gives satisfactory predictions for pitch response, while the damping in heave is somewhat overpredicted. In addition, the simulation indicates that Froude scaling is adequate for extrapolating scale-model results to full-scale craft, at least to within the accuracy indicated by the comparison of the experimental data with the simulation results.

Acknowledgment

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References

[1]Kaplan, P. and Davis, S., "A Simplified Representation of the Vertical Plane Dynamics of SES Craft," AIAA Paper, Feb. 1974.

[2]Lavis, D. R., Bartholomew, R. J., and Jones, J. C., "Response of Air Cushion Vehicles to Random Seaways and the Inherent Distortion in Scale Models," *Journal of Hydronautics*, Vol. 8, 1974, pp. 83-94.

[3]Reynolds, A. J., "A Linear Theory for the Heaving Response of a Hovercraft Moving over Regular Waves," *Journal of Mechanical Engineering Science*, Vol. 14, April 1972, pp. 147-150.

[4]Reynolds, A. J., West, R. P., and Brooks, B. E., "Heaving and Pitching Response of a Hovercraft Moving over Regular Waves," *Journal of Mechanical Engineering Science*, Vol. 14, Oct. 1972, pp. 340-352.

[5]Doctors, L. J., "Nonlinear Motion of an Air-Cushion Vehicle over Waves," *Journal of Hydronautics*, Vol. 9, April 1975, pp. 49-57.

[6]Doctors, L. J., "The Hydrodynamic Influence on the Nonlinear Motion of an Air-Cushion Vehicle over Waves," *Proceedings of the Tenth Symposium on Naval Hydrodynamics*, Office of Naval Research, Washington, 1974, pp. 389-420.

[7]Doctors, L. J., "The Effect of Air Compressibility on the Nonlinear Motion of an Air-Cushion Vehicle over Waves." Presented at the Eleventh Symposium on Naval Hydrodynamics, London, England, 1976.

[8]Carrier, R., Lundblad, W., and Swift, M. R., "The Effect of Skirt Configuration on the Seakeeping of Air Cushion Vehicles," AIAA Paper, Sept. 1976.

[9]Magnuson, A. H., "Seakeeping Trials of the BH.7 Hovercraft," Naval Ship Research and Development Center Rept. SPD-574-01, Aug. 1975.

[10]Fein, J. A., Magnuson, A. H., and Moran, D. D., "Dynamic Performance Characteristics of an Air Cushion Vehicle," *Journal of Hydronautics*, Vol. 9, Jan. 1975, pp. 13-24.

[11]Magnuson, A. H. and Messalle, R. F., "Seakeeping Characteristics of the Amphibious Assault Landing Craft," *Journal of Hydronautics*, Vol. 11, July 1977, pp. 85-92.

[12]Gear, C. W., *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice-Hall, Englewood Cliffs, N. J., 1971, pp. 87-92.